Provable Security for Program Obfuscation

Yury Lifshits

Mathematics & Mechanics Faculty
Saint Petersburg State University

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Outline

Perfect Security

What do we want to get?

We want to be sure that our system is safe to use.

In lecture 4 “Applications of Obfuscation” we’ll discuss what kind of safety we want to get by obfuscation.

Today: what does it mean to be sure about safety?

Usual approach: to build some proof of safety.

Ways to Security

How are we going to prove security?

- ⇨ Theoretic security: obfuscated program doesn’t provide enough information to successful attack
  Example: exact reverse engineering. Solution: delete comments
- ⇨ Computational (cryptographic) security: attack required too much computation

Necessary hardness of attack: average superpolynomial complexity.
Now: no problems with such proved complexity

Ways to Security II

So what can we accept as enough hard problem?

- NP-hard problems. Disadvantage: worst case complexity
- NP-hard problems with average complexity results.
  Example: SUBSET SUM
- Problems with wide-believed hardness:
  Examples: FACTORING, DISCRETE LOG

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Current Results

What are the best results to the moment?

- Specific attacks on specific programs are computationally hard
- For some classes of programs we can hide most of internal information
- Some program analysis is proved to be hard
- And obfuscation in general is impossible!

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Ana and BAna

Slide from Lecture 1 — your turn to explain.

We are interested in 2 types of polynomial-time analyzers:

- Ana is a source-code analyzer that can read the program.

$$Ana(P)$$

- BAna is a black-box analyzer that only queries the program as an oracle.

$$BAna^P(time(P))$$

Black-Box security

Ana can't get more information than BAna could

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Black-Box Security: Formal Definition

A nondeterministic algorithm O is a TM obfuscator if three following conditions hold:

- **(functionality)** For every TM M, the string $O(M)$ describes the same function as M.
- **(polynomial slowdown)** The description length and running time of $O(M)$ are at most polynomially larger than that of M.
- **("virtual black box" property)** For any PPT A, there is a PPT S and a negligible function α such that for all TMs M

$$\left| Pr[A(O(M)) = 1] - Pr[S^M(1^{|M|}) = 1] \right| \leq \alpha(|M|).$$

Black-Box Security: Two Programs

A 2-TM obfuscator is defined in the same way as a TM-obfuscator, except the "virtual black box" property is changed as follows

- **("virtual black box" property)** For any PPT A, there is a PPT S and a negligible function α such that for all TMs M and N

$$\left|Pr[A(O(M),O(N))=1]-Pr[S^{M,N}(1^{|M|+|N|})=1]\right|\le\alpha(min(|M|,|N|)).$$

What obfuscator is more powerful?

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Two Programs Lemma

Two Programs Lemma

2-TM obfuscators do not exist.

$$C_{\alpha,\beta}(x) = \begin{cases}\beta, & x=\alpha\\ 0, & \text{otherwise}\end{cases}$$

$$D_{\alpha,\beta}(C) = \begin{cases}1, & C(\alpha)=\beta\\ 0, & \text{otherwise}\end{cases}$$

$$Z_k(x) = 0^k$$

Intuition: it is difficult to distinguish pairs $C_{\alpha,\beta}, D_{\alpha,\beta}$ from pair $Z_k, D_{\alpha,\beta}$ given only black box access to these programs.

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Lemma Proof: Rough Sketch

Suppose O is 2-TM obfuscator. Let's check its "black box" property on pairs $C_{\alpha,\beta}, D_{\alpha,\beta}$ and $Z_k, D_{\alpha,\beta}$ for every α,β where $A=N(M)$.

- $$Pr[A(O(C_{\alpha,\beta}),O(D_{\alpha,\beta}))=1] = 1$$
- $$Pr[A(O(Z_k),O(D_{\alpha,\beta}))=1] = 2^{-k}$$
- $$\left|Pr[S^{C_{\alpha,\beta},D_{\alpha,\beta}}=1]-Pr[S^{Z_k,D_{\alpha,\beta}}=1]\right|\le 2^{-\Omega(k)}$$

So we get a contradiction! But...

There is a flaw in the proof. Do you see?

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Impossibility Theorem

Impossibility Theorem

TM obfuscators do not exist.

$$F_{\alpha,\beta}(b,x) = C_{\alpha,\beta}\#D_{\alpha,\beta}$$

$$G_{\alpha,\beta}(b,x) = Z_k\#D_{\alpha,\beta}$$

Algorithm A is the following: to decompose M into two parts and evaluate the second part on the code (encoding) of the first.

Argument is similar to the Lemma's proof.

Property Hiding

Slide from Lecture 1 — your turn to explain.

Instance: two families of programs Π_1 and Π_2

Adversary task: given a program $P \in \Pi_1 \cup \Pi_2$ to decide whether $P \in \Pi_1$ or $P \in \Pi_2$.

Desirable protection: make adversary task as difficult as well-known computationally hard problem is.

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Password Checking Hiding

prog π_1^w;
var x:string, y:bit;
input(x);
if $x = w$ then y:=1 else
y:=0;
output(y);
end of prog;

prog π_0;
var x:string, y:bit;
input(x);
y:=0; output(y);
end of prog;

Task: Make this families indistinguishable.

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Some Theoretical Background

One-Way Permutation is bijection from the set of all binary strings of length k to itself which is easy to compute and difficult to inverse.

$$F : B^k \rightarrow B^k$$

Hardcore Predicate for one way permutation F is a predicate (i.e. boolean function) h such that given $F(x)$ its difficult to predict $h(x)$ better than just guess it.

Usual construction of hard-core predicate: choose r by random and take any one way permutation F than given a pair $(F(x), r)$ its difficult to uncover $x \cdot r$.

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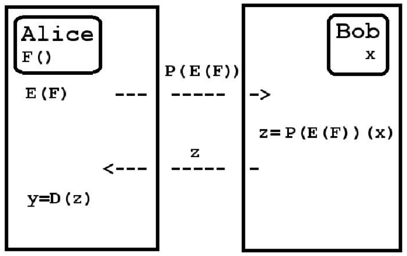
Program with hidden password checking

prog Π
var x: string, y:bit;
const u, v:string, σ:bit;
input(x);
if ONE_WAY(x)=v then
  if $x \cdot u = \sigma$ then y:=1 else y:=0;
else y:=0;
output(y);
end of prog;

Model of Encrypted Computation

Slide from Lecture 1 — your turn to explain.



<u>Basic task</u>: keep F unknown to Bob.

Homomorphic Encryption

General idea: to design an encoding such that it is possible to evaluate various operations over encrypted messages (and getting encrypted results) without decrypting them.

In particular encoding is called

- **Additively homomorphic** if it is possible to compute $E(x+y)$ from $E(x)$ and $E(y)$
- **Multiplicatively homomorphic** if it is possible to compute $E(xy)$ from $E(x)$ and $E(y)$
- **Mixed multiplicatively homomorphic** if it is possible to compute $E(xy)$ from $E(x)$ and y.

Homomorphic Encryption II

Fact: there exists additively homomorphic encryption schemes over the rings $\mathbb{Z}/N\mathbb{Z}$.

Corollary: there exists additively & mixed multiplicatively homomorphic encryption schemes over the rings $\mathbb{Z}/N\mathbb{Z}$.

Proof: Mixed multiplication could be done by polynomial number of additions.

Encrypting Polynomials

Let P be polynomial over $\mathbb{Z}/N\mathbb{Z}$ ring.

$$P = \sum a_{i_1 \dots i_s} X_1^{i_1} \dots X_s^{i_s}$$

Then we can encrypt P by just sending encrypted coefficients (using MM-A homomorphic encryption). Bob is able to compute $E(P(X))$ and return it back to Alice.

What we reveal to Bob? Only set of nonzero coefficients of P.

Mobile cryptography results

What are further results for encrypted computation?

- Other presentations of function.
  - [Loreiro, Molva] – function as a matrix.
  - [Sander, Tschudin] – another basic hard problem: decomposition of rational functions.

More Black-Box Security

What are other functions obfuscated with black-box security?

- [LPS 2004] – interactive access control system.
- Next results I expect from you!

Quality of obfuscating transformations

What is hard to get from programs after obfuscating transformations?

- Alias analysis is NP-hard!
- Average hardness is proved only for several fixed analysis algorithms

Summary

- We can prove property extracting to be hard in some cases.
- We can use cryptographic constructions to hide some internal constants.
- Obfuscation in general is impossible.

Question Time!

Not covered by the talk

- Black box security with relations to zero-knowledge