# Approximate Nearest Neighbors in Hamming Model

Algorithmic Problems Around the Web #6

Yury Lifshits
http://yury.name

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# Self-Reduction in a Nutshell

**Problem:** $(1+\varepsilon)$-approximate $l$-range queries in $d$-dimensional Hamming cube

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**Problem:** $(1+\varepsilon)$-approximate $l$-range queries in $d$-dimensional Hamming cube

- Apply embedding $\{0,1\}^d$ into $\{0,1\}^k$ such that $l$-neighbors usually fall within $\delta_1 k$ from each other, while $(1+\varepsilon)l$-far objects are embedded at least $\delta_2 k$ from each other
- Precompute all $(\frac{\delta_1+\delta_2}{2})k$-neighbors for every point in $\{0,1\}^k$
- In search step, embed $q$ and explicitly check all precomputed $(\frac{\delta_1+\delta_2}{2})k$-neighbors

# RP: Inner product test

## Single test:

- Choose random subset of positions of size $\frac{1}{2l}$
- Randomly assign 0 or 1 to every of them, the rest assign to 0, call the resulting vector $r$
- $h_r(p) = r \cdot p$

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**Claim:** there exist constants $\delta_1 > \delta_2$

- $H_d(p, s) \leq l \Rightarrow Pr[h(p) = h(q)] \geq \delta_1$
- $H_d(p, s) \geq (1 + \varepsilon)l \Rightarrow Pr[h(p) = h(q)] \leq \delta_2$

# RP: Preprocessing

**Inner product mapping:**

- Choose $k$ random tests $r_1, \ldots, r_k$
- Map every $p$ into $A(p) = h_{r_1}(p) \ldots h_{r_k}(p)$

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Data Structure

- Apply inner product mapping to all strings in database
- For every $v \in \{0,1\}^k$ precompute all $(\frac{\delta_1+\delta_2}{2})k$-neighbors

# RP: Search

- Compute $A(q) = h_{r_1}(q) \ldots h_{r_k}(q)$
- Retrieve and explicitly check all $(\frac{\delta_1+\delta_2}{2})k$-neighbors of $A(q)$

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**Analysis:**

- Chances to miss true $l$-neighbor: $\exp(-\frac{\delta_1-\delta_2}{2\delta_1}k)$
- Chances to waste time on $(1+\varepsilon)l$-far neighbor: $\exp(-\frac{\delta_1-\delta_2}{2\delta_1}k)$

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Thus we should take near-logarithmic $k$ which lead to polynomial size of $\{0,1\}^k$ to be NN-precomputed

# RP: Formal Claim

**Theorem (Kushilevetz, Ostrovsky, Rabani, 1998)**

*Consider $(1+\varepsilon)$-approximate $l$-range search in $d$-dimensional Hamming cube. Then for every $\mu$ there is a randomized algorithm with (roughly) $d^2 polylog(d, n)$ query time and $n^{\mathcal{O}(\varepsilon^{-2})}$ preprocessing space. For every query this algorithm answers correctly with probability at least $1-\mu$*

Thanks for your attention! Questions?

# References

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